Prelim Syllabus
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1 Harmonic Analysis

• Vitali and Whitney Covering Lemmas
• Interpolation: Marcinkiewicz, Riesz-Thorin. As corollaries of Riesz-Thorin: Young’s Inequality, Hausdorff-Young.
• Hardy-Littlewood Maximal Functions, Maximal Inequality, Lebesgue Differentiation as a corollary.
• Calderon-Zygmund Decomposition
• BMO. Definitions. John-Nirenberg (Journé’s proof) Duality with the atomic decomposition of $H^1$ [Stein]. Behavior on CZOs.
• Hilbert and Riesz Transforms
• Standard Estimates for Singular Integrals/Calderon-Zygmund Operators. Boundedness properties (strong $(q, q)$ implies weak $(1, 1)$).
• Pseudodifferential Operators. $L^2$ boundedness of $S^0$ class.
• $T(1)$ Theorem. Coifman-Meyer Proof.
• Cauchy Integral Operator, application of $T(1)$ theorem to it.
• Fourier Transform.
• Cotlar’s Almost Orthogonality Lemma
• Hardy-Littlewood Fractional Integration, its use to give alternate proof of Sobolev Embedding, $(1 < p < n)$

2 PDE

• Laplace and Poisson Equation: Fundamental solution, mean value formula, weak and strong maximal principles, uniqueness (via maximal principles, and via energy), Green’s representation formula [Evans 2.2]
• Heat equation: Fundamental Solution. Duhamel’s principle. Uniqueness via energy methods. [Evans 2.3]
• Wave equation: Solution in $n = 1, 2, 3$, Duhamel’s principle to get inhomogeneous solution, conservation of Energy to get uniqueness, domain of dependence [Evans 2.4]
• Gronwall’s Inequality [Evans B.2.j-k]
• Sobolev Spaces: Definitions, Gagliardo-Nirenberg Inequality, Morrey’s Inequality, Poincaré inequality. [Evans 5]

• Energy estimates for first-order symmetric hyperbolic systems [Evans 7.3]

• Banach fixed point theorem and fixed point methods [Evans 9.2]

• Derivation and assumptions for the water wave equation

References
Stein, Elias. 1970. *Singular Integrals*
Stein, Elias, 1993. *Harmonic Analysis*