

# Prelim Syllabus

Rafe Kinsey

January 10, 2011

## 1 Harmonic Analysis

- Vitali and Whitney Covering Lemmas
- Interpolation: Marcinkiewicz, Riesz-Thorin. As corollaries of Riesz-Thorin: Young's Inequality, Hausdorff-Young.
- Hardy-Littlewood Maximal Functions, Maximal Inequality, Lebesgue Differentiation as a corollary.
- Calderon-Zygmund Decomposition
- BMO. Definitions. John-Nirenberg (Journé's proof) Duality with the atomic decomposition of  $H^1$  [Stein]. Behavior on CZOs.
- Hilbert and Riesz Transforms
- Standard Estimates for Singular Integrals/Calderon-Zygmund Operators. Boundedness properties (strong  $(q, q)$  implies weak  $(1, 1)$ ).
- Pseudodifferential Operators.  $L^2$  boundedness of  $S^0$  class.
- $T(1)$  Theorem. Coifman-Meyer Proof.
- Cauchy Integral Operator, application of  $T(1)$  theorem to it.
- Fourier Transform.
- Cotlar's Almost Orthogonality Lemma
- Hardy-Littlewood Fractional Integration, its use to give alternate proof of Sobolev Embedding, ( $1 < p < n$ )

## 2 PDE

- Laplace and Poisson Equation: Fundamental solution, mean value formula, weak and strong maximal principles, uniqueness (via maximal principles, and via energy), Green's representation formula [Evans 2.2]
- Heat equation: Fundamental Solution. Duhamel's principle. Uniqueness via energy methods. [Evans 2.3]
- Wave equation: Solution in  $n = 1, 2, 3$ , Duhamel's principle to get inhomogeneous solution, conservation of Energy to get uniqueness, domain of dependence [Evans 2.4]
- Gronwall's Inequality [Evans B.2.j-k]

- Sobolev Spaces: Definitions, Gagliardo-Nirenberg Inequality, Morrey's Inequality, Poincaré inequality. [Evans 5]
- Energy estimates for first-order symmetric hyperbolic systems [Evans 7.3]
- Banach fixed point theorem and fixed point methods [Evans 9.2]
- Derivation and assumptions for the water wave equation

## References

- Christ, Michael, 1990. *Lectures on Singular Integral Operators*.
- Coifman, R.R., and Y. Meyer, 1986. "Non-linear harmonic analysis, operator theory and PDE." In *Beijing Lectures in Harmonic Analysis*, ed. Stein.
- Evans, L.C., 1998. *Partial Differential Equations*, first ed.
- Johnson, R.S., 1997. *A Modern Introduction to the Mathematical Theory of Water Waves*.
- Journé, Jean-Lin, 1983. *Calderon-Zygmund Operators, Pseudo-Differential Operators, and the Cauchy Integral of Calderon*.
- Klainerman, Sergiu. 2008. *Analysis Lecture Notes* (online).
- Stein, Elias. 1970. *Singular Integrals*
- Stein, Elias, 1993. *Harmonic Analysis*
- Tao, Terence. *Harmonic Analysis (Math 247A,B) Lecture Notes* (online).