Some Tips for Exam 2 FRIDAY, MARCH 16, 2012
MATH 116-017 GSI: RAFE KINSEY

Here's a *non-exhaustive* list of some tips that came to mind about the material. Please see the exam 1 tips (on the webpage) for general tips on studying.

Room: 1640 Chem, 6pm sharp.

Extra Office Hours: Sunday 4pm, 5832 (bring your MCard to get in). We'll be there until at least 5pm, I can stay longer than that if people have more questions. I'll also try to answer questions over email over the weekend (though give me 24 hours to respond). Note also math lab as a resource (e.g., Sunday night).

General Comments

- A **general principle**: on the exam, your job is not just to get the right answer, it's to show the grader that you fully understand the material (rather than, say, accidentally managing to get the right answer). *To get full credit, you will often need to have reasonable work.*
- Make sure to **follow instructions**. Give your answer in the form asked, don't answer more than you need, etc. (Often when the problem is subdivided in parts, it's to help you. Only answer part (a) in this part, it might then help you solve part (b).)
- **Note**: On the exam, you will be expected to compute integrals **exactly, by hand** to receive credit, unless explicitly told otherwise.
- For a lot of these problems, you have to do some *algebra*. Be careful to avoid mistakes in your algebra. I've seen a lot of you do illegal things when doing algebraic, leading you to wrong answers.
- **Don't let your graphing calculator be a crutch!** I see a lot of you spend a lot of time trying to use your graphing calculator to get an answer, when it's (a) quicker; (b) much less likely to lead you astray; and (c) much more likely to help you really understand the concepts if you do it by hand. In particular: this weekend while you study, I encourage you to avoid using your calculator as much as possible, for reason (c), since this will help you learn the material better for Monday. *It's a good habit to understand what basic functions look like without having to graph them; we don't have that many functions that we deal with, just things like polynomials, exponentials, logarithms, and various transformations of these.* Use your calculator as a way of *checking* your work, confirming what you've done. (Except of course for problems that specifically need you to calculate numerical things. But there usually aren't too many of these.)
- The exam focuses on the sections we've covered since midterm. But it's important still to be comfortable with the techniques of earlier stuff. *In particular, make sure you are good at integrating, by parts and by substitution.* A lot of the exam (and quiz) problems require you to calculate integrals exactly. I've seen a lot of you make mistakes with this (and even with basic anti-derivatives). So make sure to review these. The practice gateway exams are still available online if you want practice. Remember: if you've tried one of substitution or integration-by-parts and it looks messy, try the other one (or try making a different choice of substitution, etc.).
- A lot of problems ask you to solve for something, when you have certain information. Make sure you write down the information you have from the problem, in terms of the things you know from the material. Then it's often a matter of doing easier algebra or Calc I stuff that you should already know.

It's a good idea to first make sure that you feel comfortable with the material. Making study notes is a good way of seeing if you feel comfortable with the material. Go over the sections first, do some practice problems from the book. Practice true and false questions from the *check your understanding section.* Once you feel comfortable with the material, do old exam problems (or redo quiz problems, which are usually from old exams).[1] **Make sure to do problems without looking at the solutions. After you've worked on it, look at the solutions. Then put the solutions away and try to do it again.**

It's a good idea to do a practice exam in a timed 90 minutes, to give you practice in managing your time well.

Differential Equations

- Remember: differential equations have a *family of solutions.* If you specify a specific point, that tells you which specific solution. Often, you will be told something which determines which specific solution this is. In this case, you have to carefully use the given information to solve for the unspecified constant in the family of solutions. To do this, you *have to be careful*; see quiz 6, 1b for an example of this, it's not always just replacing the constant with the initial condition.
- For slope fields, make sure to pay attention to whether it depends on x, y, both; whether it's periodic (in x or y), where it's 0, etc.
- The idea of slope fields is to give you a *qualitative* idea of what the family of solutions look like.
- Remember how to do Euler's method (if there's a question like this, it should be easy—it's a simple calculation).
- For separation of variables: make sure to be careful to consider separately the case where you'd be dividing by zero. Be careful with which constants are positive, negative, etc., and remember the absolute value for ln. See page 565 of the book for this.
- *Be careful with integration* when you do separation of variables. What is $\int \frac{dy}{5y-6}$? It's *not* just $\ln|5y-6|$.
- When you are modeling things the process is: interpret the information to get a differential equation, then solve it (or do something with it). Remember, we think of continuous interest as $\frac{dy}{dt} = ky$.[2] Sometimes we have multiple things happen: continuous growth at rate k, but then a fixed amount j taken out per year: then we would get $\frac{dy}{dt} = ky - j$.
- Be careful to keep straight the difference between a differential equation and a solution. If something has a constant rate of change, we have $dy/dt = k$ (not kt, as I saw some of you incorrectly write!) but $y = kt + C$.
- Be careful when you model something with a differential equation to pay attention to the signs of your parameters. If you say $\frac{dy}{dt} = ky$, is k positive or negative? Sometimes you have the choice of how to write the constant of proportionality, but *if you're told in the problem what it is, you have to follow that convention.*

Parametric and Polar

(These are similar; the comments below, if given for parametric, might apply to polar as well, and vice versa.)

[1]You can find old exams on the course website. (Note that a few of the old exams might have problems from different sections. The most recent exams should follow the same schedule as us.) There's also a special webpage http://dhsp.math.lsa.umich.edu/ mconger/dhsp/116exams/116exam2shop.html which allows you to choose specific problems of a given type (and has older exams). If you use this, I'd recommend downloading the problems earlier, in case the site crashes.

[2]If you're looking for a differential equation, we think of it in these terms, instead of Ce^{kt}. This is important when there's something else that's happening.

- Work step by step for these. The techniques we use for these are often basic. If you want when $x = 0$, solve for $x(t) = 0$.[3] **Note**: you need to do a little bit of work to justify this or similar things, or else you might not get credit.[4]

- If you want when y reaches a maximum, solve $y'(t) = 0$, and then decide which of these critical points gives you the maximum. *You need to consider all possible critical points*, and show that you've done some work (e.g., looking at the graph) to show which ones are the actual maxima/minima you are looking for.

- Make sure to have on your index card the various formulas for second derivatives, arclength, etc.

- You're responsible for knowing the formula for arc length from problem 41 on page 415 in 8.4.

- For polar coordinates, pay *very close* attention to which range of θ is relevant. Note that some curves might go all the way around from 0 to π instead of 0 to 2π (or more complicated things). *To figure this out, you should try a bunch of points out by hand.*[5] Try things like $\theta = 0, \pi/6, \pi/4, \pi/3, \pi/2$, etc. Be careful of when the radius is negative.

- Remember that inequalities describing regions in polar coordinates will have r between functions of θ, like in the quiz problem.

Improper Integrals

For 7.7:

- To calculate improper integrals exactly **don't forget to include the limit part in every line of your string of equalities**! Make sure not to skip this step, you'll lose points. Once you've done this, the limits of definite integrals you should calculate *using the tools you know from the first exam*: anti-derivatives, substitution, integration-by-parts.

- Be careful with substitution: **you have to change the limits of integration in the definite integrals**!

- To calculate limits: there are only a few things you'll need to be able to do, things like $\lim_{b\to\infty}$ of $b^p, e^{ap}, e^{-ap}, \ln(\pm b)$, or similarly as $b \to 1, b \to 0, b \to -\infty$. It's important to know that exponentials are more powerful than polynomials: xe^{-x} will go to 0 as $x \to \infty$. A good way of seeing what happens for these functions is to think about their graphs. You can also see section 1.8 on this.

For 7.8:

- Remember the process for the comparison test:
  - Step 1: Guess whether the integral converges or diverges, by finding another function that is similar to the integrand *near where the problem is.* For example: if you have $\int_3^\infty \frac{3+\sin x}{x^3+8}dx$, the problem is as $x \to \infty$, so the function $\frac{3}{x^3}$ will be similar to the function $\frac{3+\sin x}{x^3+8}$ as $x \to \infty$; for $\int_5^9 \frac{1}{(t-5)^2+3}dt$ the problem is as $t \to 5$, so you want something similar as $t \to 5$, and so $\frac{1}{(t-5)^2}$ is a good comparison.[6]

[3] Be careful, something crosses the x axis when $y = 0$.

[4] E.g., if a problem says, "Find what times the thing crosses the x-axis, where $x(t) = \sin t, y(t) = (t-6)(t-3)t^2$" you should "We want when $0 = y(t) = (t-6)(t-3)t^2$, so $t = 6, 3, 0$ are the solutions." If you just write $t = 6, 3, 0$, you might not get credit.

[5] Do this in addition to using the calculator. Use the calculator to get a quick picture, and then trace out several points by hand.

[6] A good way of phrasing it is as I have done "as $x \to$[something, e.g. ∞ or 5], the [original function] is like [the new function]."

- Then figure out the convergence/divergence of the integral of the other function, remembering the convergence/divergence of things like $\int_0^1 \frac{1}{x^p}dx$ and $\int_1^\infty \frac{1}{x^p}dx$, as well as $\int_0^\infty e^{-ax}dx$. **Be careful that you have this down correctly, note the difference between the integrals from** 0 **to** 1 **and the integrals from** 1 **and** ∞**.**[7] Be careful about notation, some books might write it in a slightly different form.
- Now you **have to** to step 2. If you think the integral converges, then you need to find a *larger* comparison function whose integral also converges. (Note that the comparison function has to be positive; see my comment on quiz08-solns.) If you think the integral diverges, then you need to find a *smaller* comparison function whose integral also diverges.
- The obvious first choice is the function you considered in the previous step. Try this out—see whether it satisfies the *correct* inequality. (This is an inequality of functions, e.g. $\frac{1}{x^2+1} \leqslant \frac{1}{x^2}$ for all x.) If it does, make sure to write down that it does satisfy the inequality.
- If the obvious function doesn't satisfy the correct inequality, modify it slightly. The trick that usually works is: multiply it by 2 (or 100) to make it bigger if it's too small, multiply it by $\frac{1}{2}$ (or $\frac{1}{100}$) if it's too big. Then check to see if that satisfies the inequality. Often you'll do this by cross-multiplying, and you'll see that it works for some range of x. Remember the principle in the box on page 383; as long as the range where it works includes where "bad things" happen (e.g., as $x \to \infty$ or $x \to 0$), then the convergence of the integral just depends on the convergence of the integral in that range.
- **Double-check** that you've done the right inequality for the comparison test. State that the integral of the comparison function converges/diverges. (You can just quote the facts on page 382, you don't need to show them. But make sure to do them correctly!)

- Be careful about your language: **say that an integral converges, don't say that a function converges**. E.g., say "and we know that $\int_1^\infty \frac{1}{x^2}dx$ converges" but don't say ~~"and we know that $1/x^2$ converges"~~.
- For integrals like $\int_5^9 \frac{1}{\sqrt{t-5}}dt$, making a substitution like $u = t - 5$ will turn these into integrals like $\int_0^c \frac{1}{\sqrt{u}}du$. (To do the substitution carefully, you should use the limits, since it's an improper integral.)
- Remember that what matters most, in step 1, is the most important term of the numerator and denominator of rational functions (i.e., functions that are ratios of polynomials). As $x \to \infty$, those are the highest-order terms. As $x \to 0$, these are the *lowest-order terms* (like on the tricky problem in the quiz).
- I strongly recommend you do more practice of these problems, making sure that you've written up the answer carefully. Look at the solutions that I wrote up for the optional practice hw problems (posted in the handouts section of webpage).

Probability

- Remember, the equation for mean is $\int_{-\infty}^\infty xp(x)dx$, don't include any denominator.
- Make sure to be careful about upper-vs-lower-case letters $P(x)$ vs $p(x)$.
- Don't forget the condition $\int_{-\infty}^\infty p(x)dx = 1$. This comes up naturally in exam-type problems.
- Remember the relationship $P'(x) = p(x)$.

Good luck! Make sure to get a good night's sleep and take care of yourself before the exam (not just the night before, but two nights before).

[7]Note: if you forget them, **you should be able to figure it out yourself**, it's a 7.7-level problem to calculate them yourself.